token	t, u, v, w	::=	*string*	
associativity	a	::=	L	(left assoc)
		|	R	(right assoc)
precedence	n, m	::=	$\mathbb{N}$	
fixity	*fix*	::=	atom	(has no children)
		|	$\text{prefix}_a(n)$	
		|	$\text{suffix}_a(n)$	
		|	$\text{infix}_a(n)$	
rule	r	::=	t *fix*	
grammar	G	::=	$\overline{r}$	
tree	x, y, z	::=	$(\overline{x}t)$	(tree in RPN)
state	s	::=	$\overline{x} \left\langle \overline{t} \right\rangle \overline{t}$	(parsing state)
		|	$\overline{x} \, ? \left\langle \overline{t} \right\rangle \overline{t}$	(parsing state w/ hole)

Precedence rules:

$$\boxed{G \vdash n \lessdot t \qquad G \vdash t \gtrdot n \qquad G \vdash t \lessdot n \qquad G \vdash n \gtrdot t \qquad G \vdash \emptyset \lessdot t \qquad G \vdash t \gtrdot \emptyset}$$

$$\frac{t \text{ atom} \in G}{\begin{array}{l} G \vdash t \lessdot 0 \\ G \vdash 0 \gtrdot t \end{array}} \qquad \frac{t \text{ infix}_L(n) \in G}{\begin{array}{l} G \vdash n \lessdot t \\ G \vdash t \gtrdot n - 1 \\ G \vdash t \lessdot n \\ G \vdash n \gtrdot t \end{array}} \qquad \frac{t \text{ infix}_L(n) \in G}{\begin{array}{l} G \vdash n - 1 \lessdot t \\ G \vdash t \gtrdot n \\ G \vdash t \lessdot n \\ G \vdash n \gtrdot t \end{array}}$$

$$\frac{t \text{ prefix}_L(n) \in G}{\begin{array}{l} G \vdash \emptyset \lessdot t \\ G \vdash t \gtrdot n - 1 \\ G \vdash t \lessdot n \\ G \vdash 0 \gtrdot t \end{array}} \qquad \frac{t \text{ prefix}_R(n) \in G}{\begin{array}{l} G \vdash \emptyset \lessdot t \\ G \vdash t \gtrdot n \\ G \vdash t \lessdot n \\ G \vdash 0 \gtrdot t \end{array}} \qquad \frac{t \text{ suffix}_L(n) \in G}{\begin{array}{l} G \vdash n \lessdot t \\ G \vdash t \gtrdot \emptyset \\ G \vdash t \lessdot 0 \\ G \vdash n \gtrdot t \end{array}} \qquad \frac{t \text{ suffix}_R(n) \in G}{\begin{array}{l} G \vdash n - 1 \lessdot t \\ G \vdash t \gtrdot \emptyset \\ G \vdash t \lessdot 0 \\ G \vdash n \gtrdot t \end{array}}$$

$$\frac{G \vdash t \lessdot n \quad n \leq m}{G \vdash t \lessdot m} \qquad \frac{G \vdash t \gtrdot n \quad n \geq m}{G \vdash t \gtrdot m}$$

$$\boxed{G \vdash t \lessdot t}$$

$$\frac{G \vdash t \lessdot n \quad G \vdash n \lessdot u}{G \vdash t \lessdot u} \qquad \frac{G \vdash t \gtrdot n \quad G \vdash n \gtrdot u}{G \vdash t \gtrdot u}$$

$$\boxed{G \vdash x}$$

$$\text{T-Atom}\frac{t \text{ atom} \in G}{G \vdash (t)} \qquad \text{T-Infix}\frac{\begin{array}{c} t \text{ infix} \in G \\ G \vdash x \quad G \vdash \mathit{head}(x) \lessdot t \\ G \vdash y \quad G \vdash t \gtrdot \mathit{head}(y) \end{array}}{G \vdash (x\, t)}$$

$$\text{T-Prefix}\frac{\begin{array}{c} t \text{ prefix} \in G \\ G \vdash x \quad G \vdash t \gtrdot \mathit{head}(x) \end{array}}{G \vdash (x\, t)} \qquad \text{T-Suffix}\frac{\begin{array}{c} t \text{ suffix} \in G \\ G \vdash x \quad G \vdash \mathit{head}(x) \lessdot t \end{array}}{G \vdash (x\, t)}$$

Figure 1: Precedence Rules and validity of parse trees

$\boxed{G \vdash \overline{x}\left\langle\, \overline{t} \,\right\rangle \overline{t}}$

$$\frac{G \vdash x}{G \vdash x\left\langle\ \right\rangle \overline{w}} \qquad \frac{\begin{array}{c} u \text{ prefix} \in G \\ G \vdash y \qquad G \vdash u \gg \mathit{head}(y) \\ G \vdash \overline{x}\, 0 \left\langle\, \overline{t} \,\right\rangle \overline{w} \\ G \vdash y \lessdot \mathit{first}(\overline{w}) \ \mathit{or}\ \overline{w} = \epsilon \end{array}}{G \vdash \overline{x}\, y \left\langle\, \overline{t}\, u \,\right\rangle \overline{w}} \qquad \frac{\begin{array}{c} u \text{ infix} \in G \\ G \vdash y \qquad G \vdash \mathit{head}(y) \lessdot u \\ G \vdash z \qquad G \vdash u \gg \mathit{head}(z) \\ G \vdash n \gg u \qquad G \vdash \overline{x}\, n \left\langle\, \overline{t} \,\right\rangle \overline{w} \\ G \vdash z \lessdot \mathit{first}(\overline{w}) \ \mathit{or}\ \overline{w} = \epsilon \end{array}}{G \vdash \overline{x}\, y\, z \left\langle\, \overline{t}\, u \,\right\rangle \overline{w}}$$

$\boxed{G \vdash \overline{x}\, n \left\langle\, \overline{t} \,\right\rangle \overline{t}}$

$$\frac{}{G \vdash\ n\left\langle\ \right\rangle} \qquad \frac{\begin{array}{c} u \text{ prefix} \in G \qquad G \vdash u \gg m \\ G \vdash \overline{x}\, 0 \left\langle\, \overline{t} \,\right\rangle \overline{w} \end{array}}{G \vdash \overline{x}\, m \left\langle\, \overline{t}\, u \,\right\rangle \overline{w}} \qquad \frac{\begin{array}{c} u \text{ infix} \in G \qquad G \vdash u \gg m \\ G \vdash y \qquad G \vdash \mathit{head}(y) \lessdot u \\ G \vdash n \gg u \qquad G \vdash \overline{x}\, n \left\langle\, \overline{t} \,\right\rangle \overline{w} \end{array}}{G \vdash \overline{x}\, y\, m \left\langle\, \overline{t}\, u \,\right\rangle \overline{w}}$$

Figure 2: Validity of parsing states

$\boxed{G \vdash s \to s}$

$$\text{P-Atom}\frac{v \text{ atom} \in G}{G \vdash \overline{x}\, ?\left\langle\, \overline{t} \,\right\rangle v\, \overline{w} \to \overline{x}\, v \left\langle\, \overline{t} \,\right\rangle \overline{w}} \qquad \text{P-Suffix}\frac{\begin{array}{c} v \text{ suffix} \in G \\ G \vdash \mathit{last}(\overline{t}) \gg v \ \mathit{or}\ \overline{t} = \epsilon \end{array}}{G \vdash \overline{x}\, y \left\langle\, \overline{t} \,\right\rangle v\, \overline{w} \to \overline{x}\, (y\, v) \left\langle\, \overline{t} \,\right\rangle \overline{w}}$$

$$\text{P-PushPrefix}\frac{v \text{ prefix} \in G}{G \vdash \overline{x}\, ?\left\langle\, \overline{t} \,\right\rangle v\, \overline{w} \to \overline{x}\, ?\left\langle\, \overline{t}\, v \,\right\rangle \overline{w}} \qquad \text{P-PushInfix}\frac{\begin{array}{c} v \text{ infix} \in G \\ G \vdash \mathit{last}(\overline{t}) \gg v \ \mathit{or}\ \overline{t} = \epsilon \end{array}}{G \vdash \overline{x}\left\langle\, \overline{t} \,\right\rangle v\, \overline{w} \to \overline{x}\, ?\left\langle\, \overline{t}\, v \,\right\rangle \overline{w}}$$

$$\text{P-PopPrefix}\frac{\begin{array}{c} u \text{ prefix} \in G \\ G \vdash u \lessdot \mathit{first}(\overline{w}) \ \mathit{or}\ \overline{w} = \epsilon \end{array}}{G \vdash \overline{x}\, y \left\langle\, \overline{t}\, u \,\right\rangle \overline{w} \to \overline{x}\, (y\, u) \left\langle\, \overline{t} \,\right\rangle \overline{w}} \qquad \text{P-PopInfix}\frac{\begin{array}{c} u \text{ infix} \in G \\ G \vdash u \lessdot \mathit{first}(\overline{w}) \ \mathit{or}\ \overline{w} = \epsilon \end{array}}{G \vdash \overline{x}\, y\, z \left\langle\, \overline{t}\, u \,\right\rangle \overline{w} \to \overline{x}\, (y\, z\, u) \left\langle\, \overline{t} \,\right\rangle \overline{w}}$$

Figure 3: Parsing algorithm

```
function parse(ts):
  let (x, vs) = parseRightArg(top, ts) in
  assert vs is empty
  return (x, vs)

function parseRightArg(u, v ts):
  case v of
    atom => parseSuffix(u, (v), ts)
    prefix => let (x, ts') = parseRightArg(v, ts)
              parseSuffix(u, (x v), ts')

function parseSuffix(u, x, ts):
  case ts of
    empty => (x, empty)
    v ts =>
      if u *> v and x <* v
      then case v of
        suffix => parseSuffix(u, (x v), ts)
        infix => let (y, ts') = parseRightArg(v, ts)
                 parseSuffix(u, (x y v), ts')
      else (x, v ts)
```

Figure 4: Parsing algorithm (code)