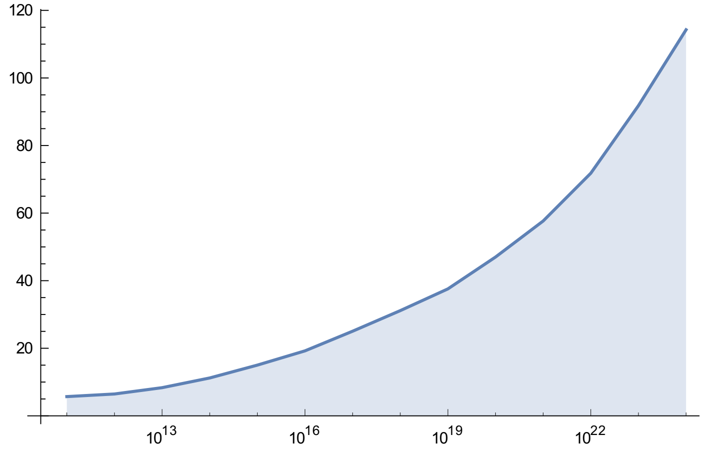
```
In[52]:= (* List of fast Gourdon alpha factors (alpha = alpha_y * alpha_z) found
          by running pi(x) benchmarks using the find_optimal_alpha_y.sh script *)

         alphaGourdon = {{10^11, 5.694}, {10^12, 6.470}, {10^13, 8.336}, {10^14, 11.210},
           {10^15, 15.016}, {10^16, 19.231}, {10^17, 25.050}, {10^18, 31.139}, {10^19, 37.573},
           {10^20, 47}, {10^21, 57.670}, {10^22, 71.804}, {10^23, 91.799}, {10^24, 114.265}}

Out[52]= {{100 000 000 000, 5.694}, {1 000 000 000 000, 6.47}, {10 000 000 000 000, 8.336},
          {100 000 000 000 000, 11.21}, {1 000 000 000 000 000, 15.016}, {10 000 000 000 000 000, 19.231},
          {100 000 000 000 000 000, 25.05}, {1 000 000 000 000 000 000, 31.139},
          {10 000 000 000 000 000 000, 37.573}, {100 000 000 000 000 000 000, 47},
          {1 000 000 000 000 000 000 000, 57.67}, {10 000 000 000 000 000 000 000, 71.804},
          {100 000 000 000 000 000 000 000, 91.799}, {1 000 000 000 000 000 000 000 000, 114.265}}

In[53]:= ListLogLinearPlot[alphaGourdon, Filling → Bottom, Joined → True]
```

Out[53]=



```
In[55]:=
         (* alpha is a tuning factor that balances the compuation of the easy special leaves
          (A + C formulas) and the hard special leaves (D formula). The formula below is used in
          the file src/common.cpp to calculate a fast alpha factor for the computation of pi(x). *)

         NonlinearModelFit[alphaGourdon, a (Log[x])^3 + b (Log[x])^2 + c Log[x] + d, {a, b, c, d}, x]

Out[55]= FittedModel[ -148.127 + 13.6067 Log[x] - 0.41743 ≪1≫^2 + 0.00464541 Log[x]^3 ]
```